## Technion - Israel Institute of Technology

The William Davidson Faculty of Industrial Engineering \& Management

# Center for Service Enterprise Engineering (SEE) 

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## SEEStat 3.0 Tutorial

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## Introduction

SEEStat is a system for Exploratory Data Analysis (EDA) in real-time. It enables users to easily conduct statistical and performance analyses of massive datasets; in particular, analyzing datasets representing operational histories of large service operations (e.g. call centers, hospitals, internet sites), such as those available through the SEELab server. SEEStat can also automatically create sophisticated reports in Microsoft Excel, which support research and teaching.

Both SEEStat and the SEELab Server were developed at the Faculty of Industrial Engineering and Management, Technion, Israel Institute of Technology. More information on the SEELab can be found at its homepage https://seelab.net.technion.ac.il/.

How to connecting to SEEStat on the Technion SEELab Server can be found here.

## SEEStat Tutorial

## USBank Data

Background: The source of this example database is a large call center of a U.S. Bank. This call center has sites in 4 states, which are integrated to form a single virtual call center: Calls are queued up, when appropriate, in a central queue; they are then served by agents across sites, by fitting service types to agent skills using SBR (Skills-Based Routing) algorithms.
The virtual call center has about 900-1200 agent positions on weekdays, and 200-500 agent positions on weekends. Agents process up to 300,000 calls per day (about $20 \%$ reach the agentqueue, and the rest complete their service process within the VRU = Voice Response Unit).


## Customer flow chart of the USBank call center

The following flow-chart describes the process-flow of calls in a typical day (Tuesday, April 2, 2002). There are 4 entry points to the system: through the VRU (Voice Response Unit), Announcement, Message, and Direct group (callers that directly connect to an agent). The most commonly used is the VRU. Most of the calls end service in the VRU (196143 calls about $80 \%$ of all calls); while around $20 \%$ of the callers entering the system seek service by an agent ('Offered Volume'). Less than 1\% of the Offered Volume calls will not reach an agent service - those customers abandon the queue while waiting (a few are disconnected due to technical problems - 'Cancel'). When an appropriate agent becomes available the customer is getting served ('Handled'), after which the customer either exits the system or is moved to a secondary service ('Continued').


## Part 1

After connecting to the server, click the SEEStat 3.0 icon to open the program.
On the top of the screen you see the main menu. Click "Main". We shall work with "Statistical Models (Summaries)". Click it.


A list-box with SEEStat studies appears (three databases in our case). Select USBank (the database we shall be working with), click "OK" and wait a few seconds.

Now you see the "Model" panel.


## Example 1.1: Distributions

We shall now create a histogram of the service time (duration) distribution, a 1-second resolution.
Click the "Distributions" button. Three available distribution models appear. Select "Estimates".


You see the tab control that has 4 tabs:
"Variables", "Options", "Select Categories" and "X Properties".


The first one "Variables" is active. This tab is mandatory, which means you must select variable(s) before moving forward. The three other tabs are optional, which means that they already have default values.

NOTE: You can select (click) several variables simultaneously by holding the Ctrl key and clicking on the variables one by one.

Select "Agent service time"(the last entry in the list).

| Variables | Options | Select Categories | $\times$ Properties |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| Variable |  |  |  | $\times$ Type |
| NIQ delay |  |  |  | Duration |
| VRU time (offered) |  |  |  | Duration |
| Customer service time |  |  |  | Duration |
| Shift duration |  |  |  | Duration |
| VRU only time |  |  |  | Duration |
| Entry time(offered) |  |  |  | Duration |
| Wait time(all) |  |  |  | Duration |
| Wait time(waiting) |  |  |  | Duration |
| Wait time(short abandons] |  |  |  | Duration |
| Wait time[abandons] |  |  |  | Duration |
| Wait time(other unhandled) |  |  |  | Duration |
| Wait time(unhandled) |  |  |  | Duration |
| Wait time(handled) |  |  |  | Duration |
| Agent service time |  |  |  | Duration |

Now move to the "Select Categories" tab. You see a list box with all the service types that are offered by USBank. Select "Retail", which is the Bank’s main service.

```
Select Categories X Properties
```

```
service
Total
Retail
Premier
Business
Platinum
```

Open the "X Properties" tab. It is used to set properties of charts and tables. On the left side you can see the "Resolution" list box. The default resolution (bin-size of the histogram) of 5 seconds is marked. Select the minimal resolution 00:01 $=1$ second, in order not to miss any details of the histogram.


Now you must select the dates we focus on. Click the "Dates ->" button on the right side.


You see the list of months for which the requested data is available.
Select "April 2001".
Below the list of months, you see two options for date-selection (Date type): "Aggregated days" and "Individual days". "Aggregated days" is the chosen-default, which we now follow.

| Months | Days |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  | Month | Year | $\wedge$ |
|  |  | March | 2001 |  |
|  |  | April | 2001 |  |
|  |  | May | 2001 |  |
|  |  | June | 2001 |  |
|  |  | July | 2001 | $v$ |
|  |  | August | 2001 |  |
|  |  | September | 2001 |  |
|  |  | October | 2001 |  |
|  |  | November | 2001 |  |
|  |  | December | 2001 |  |
|  |  | January | 2002 |  |
|  |  | February | 2002 |  |
|  |  |  | 3002 |  |
| Days type |  |  |  |  |
|  | Ondividual days |  | () Aggregated days |  |

Click "Days" to make the selection of days, and select "Weekdays" - an aggregation of all 5 working days of the week. (Holidays and some special days, such as when there is a system failure, are excluded.)


All selections have now been completed: click "OK" at the bottom right.


Wait a few seconds - SEEStat is processing your request: you now see the chart/histogram, produced as "Chart 1" within an Excel spreadsheet.
NOTE: All the examples in this tutorial, from now on, will be accumulated in this Excel file. DO NOT modify or close this Excel file.

Looking at the chart, you see some irregularities on the left (near the origin). We shall look at these more carefully later.


In fact, two sheets have been created: The first is the chart in "Chart1"; the second is "Table1", which includes Table(s) that are associated with the chart, with the default one being the "Statistics" table. Click "Table1" to see the contents of this table ( $N=619,096$ is the number of observations); skim through the summary data and then return to "Chart1".

| Statistics |  |
| :--- | ---: |
|  | Agent <br> service <br> time |
| N | 619096 |
| N(average per day) | 30954.8 |
| Mean | 4 min 19 sec |
| Standard Deviation | 4 min 35 sec |
| Variance | 21 min 2 |
| Median | 2 min 57 sec |
| Minimum | 0 |
| Maximum | 59 min 53 sec |
| Skewness | 3.062 |
| Kurtosis | 15.38 |
| Standard Error Mean | 0 sec |
| Interquartile Range | 3 min 53 sec |
| Mean Absolute Deviation | 1 min 42 sec |
| Median Absolute Deviation(MAD) | 106.17 |
| Coefficient of Variation (CV) (\%) | 2 min 6 sec |
| L-moment 2 (half of Gini's Mean Difference) | 0.383 |
| L-Skewness | 0.245 |
| L-Kurtosis | 48.57 |
| Coefficient of L-variation (L-CV) (\%) <br> (Gini's Coefficient) |  |

You can easily make modifications to charts and tables, as long as they do not require the loading of new data from the database. You will now go through an example of such a modification.

First, return to the SEEStat main menu by clicking the SEEStat USBank button, on the task bar on the lower-left side of the screen - you will repeat this action each time you wish to transfer from Excel to SEEStat.


Click "Output" on the right side of the top main menu; after this click "Modify Tables and Charts"

SEESTAT USBank - [Excel]


Two tabs are available: "Options" and "Properties". Open "Properties" and change the resolution to 00:10 = 10 seconds.

| Options | Properties |
| :---: | :---: |
|  | Resolution |
|  | $\begin{aligned} & 10: 01 \\ & 10: 02 \\ & 10: 03 \\ & 10: 04 \\ & 10: 05 \\ & 10: 06 \\ & 10: 08 \\ & 10: 09 \end{aligned}$ |
|  | 0:10 |
|  | $\begin{aligned} & 10: 12 \\ & 10: 15 \end{aligned}$ |

Click "OK".


The chart is becoming smoother, but at the cost of losing some details on the left, near the origin.

## Example 1.2: Intraday time series

We now create a chart of arrival-counts to the call center(s) of USBank, during several days in a September.

First you must return to the "Statistical Models (Summaries)" window. Click the SEEStat button on the task bar (left-bottom), next click "Windows" on the main menu (at the top) and select "Statistical Models (Summaries)"
-

| Main | - View | Findows | 近沮Output |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  | 1 Excel <br> 2 Statistical Models (Summaries) |  |

We are now changing models. To this end, select the "New Model" button (right side).


Select now "Time Series" and then select "Intraday".


As in Example 1.1, four tabs appear. In the "Variable" tab, select "Arrivals to queue". In the "Select Categories" tab, select "Total".

| Variables | Options | Select Categories | $\times$ Properties |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Variable |  |  |  |
| Agents on line |  |  |  |
| Agents on line MS |  |  |  |
| Agent status |  |  |  |
| Agent status MS |  |  |  |
| Average agents in system |  |  |  |
| Customers served and in queue (average) |  |  |  |
| Customers in system [average) |  |  |  |
| Offered load |  |  |  |
| Arivals to queue |  |  |  |
| Abandons |  |  |  |
| Abandons proportion |  |  |  |

Now select dates: Click "Dates ->"; Select September 2001 from the "Months" list; Mark "Individual days", and click the "Days" button.


The list of days contains the date, the day of the week and comments if any. For example, Monday, September $3^{\text {rd }}$, was Labor Day.
It is expected that the Tuesday following a holiday will be a busy day. We thus compare all Tuesdays of the month: September 4, September 11, September 18 and September 25.

Hold down the "Ctrl" key, and in parallel click, one by one, the four Tuesdays of September 2001.

| Days |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Day | Month | Year | Week Day | Comments |
| 1 | September | 2001 | Saturday |  |
| 2 | September | 2001 | Sunday |  |
| 3 | September | 2001 | Monday | Labor Day |
| 4 | September | 2001 | Tuesday |  |
| 5 | September | 2001 | Wednesday |  |
| 6 | September | 2001 | Thursday |  |
| 7 | September | 2001 | Friday |  |
| 8 | September | 2001 | Saturday |  |
| 9 | September | 2001 | Sunday | ShutDown from 6:30 |
| 10 | September | 2001 | Monday |  |
| 11 | September | 2001 | Tuesday |  |
| 12 | September | 2001 | Wednesday |  |

Then click "OK" (bottom right).


Note: The graphs appear in "Chart2" of Excel. As before,"Table2" contains the corresponding numerical data.

You see a sharp drop in the number of calls around 09:00 a.m. on September 11, 2001 - this is of course not surprising, given that one of the call centers of US Bank was in NYC, and the others located on the U.S. East Coast.
You also see that the Tuesday after Labor Day (4.9) is indeed a heavily-loaded Tuesday, as anticipated.
The chart is noisy, due to its 5-minute resolution. We shall momentarily increase the resolution to 1 hour ( 60 minutes). We also note the following:
On the two Tuesdays after September 11, the number of calls is low, relative to the Tuesday after Labor Day. A natural question now arises: Is there a "shape of a Tuesday"? To seek a common pattern for (the shape of) a Tuesday, if there is any, we change the graphs from absolute counts to "percent to mean" (mean = average number of calls per resolution period).

Go back to the main menu via the SEEStat tab (bottom-left). In the main menu select "Output" then "Modify Tables and Charts". In the "Options" tab, under the "Convert to" table on the left, select "Percent to mean", and in the "Properties" tab set resolution to 60:00 = 1 hour,


Click "OK".


The "Shape of a Tuesday" is clearly manifested: the distribution of calls over the day is almost the same for the three Tuesdays, both normal and heavily-loaded. (Surprisingly, September 11 also catches up from around 13:00 or so.) For example, the arrival rate during the peak hour-from 10:00-11:00—is about 2.5 times that of an average hour.

Instead of "Percent to mean", you can plot according to "Proportion to column totals" which, in simple words, means the "hourly fraction of load":

Going via the "SEEStat" tab, "Output", "Modify Tables and Charts", "Proportions to column totals", and then "OK".


You see that the arrivals during the peak hour 10:00-11:00 constitute 10\% of the daily total. (Such observations make load-predictions much easier: indeed, only the daily total must be predicted. Once the daily total is determined, the number of arrivals per hour is allocated according to the shape of the day; e.g. 10\% allocated to 10:00-11:00.)

## Example 1.3: Time series (Daily totals)

There are two types of daily-total time-series: individual days during a specific month and aggregated days by months. We now demonstrate these concepts.

Click the SEEStat button on the task bar (left-bottom); next click "Windows" on the main menu (at the top) and select "Statistical Models (Summaries)". Click the "New Model" button. Select "Time Series", then "Daily totals".


From the variables list select "Arrivals to offered" (around the middle of the list - it counts incoming calls that reached the queue for an agent service).
Click the "Dates->" button.


Mark "Days for one month" and select (after scrolling down) February 2003.


Open the "Days" tab (there is no need for you to select anything, but note the Comments). Click "OK".


We first observe the weekly pattern in which weekdays have much higher arrivals than weekends (weekends are marked blue).
Note also that on February 12, the system stopped working at 4:00 PM, and February 17 was a holiday-Washington's birthday. This is manifested on the chart, where these special days are marked as Abnormal (green) and Holiday (red).

Return to the "Statistical Models (Summaries)" window via the SEEStat tab.
(A reminder: Click the SEEStat button on the task bar (left-bottom), click "Windows" on the main menu (at the top) and select "Statistical Models (Summaries)").
Click the "<-Tables" button (top right).


From the variables list select "Number of agents" (the first option).
Open the "Select Categories" tab. Select the following three services: "Premier" (priority Retail service) "Subanco" (Spanish language) and "Quick\&Reilly" (brokerage). (In order to do so, hold down the Ctrl key and click the three options, one by one.)

Now click the "Dates->" button. Mark "Aggregated days by months" and click "Select all".
Open the "Days" tab and select "Weekdays".
Click "OK".


You see that one of the selected services (Quick\&Reilly) was integrated into the Call Center of USBank only in November 2002.

## Part 2

## Example 2.1: Distribution fitting

We now fit a parametric service-time distribution to the service-time data from Example 1.1
Open window "Statistical Models (Summaries)". Click "New Model" and select
"Distributions" and "Fitting".


From the variables list select "Agent service time".
Open the "Options" tab. You see the list of distributions available for fitting. Mark simultaneously 3 of them: Exponential, Lognormal, and Lognormal (Shifted). Set chart type to "Polygon".


Open the "X Properties" tab and set resolution to 00:01 $=1$ second.
Click the "Dates->" button. Select April 2001 and "Aggregated days", open the "Days" tab and select "Weekdays".
Click the "<-Tables" button.
On the "Select Categories" tab select Retail.
Click "OK".


Observe again the irregularities near the origin. It looks as though there are at least three distributions involved: very short calls, abnormally short calls and, after around 30 seconds, the pattern looks rather regular. The best fit is produced by the Lognormal (Shifted) distribution, but clearly, close to the origin from the right, the fit is inadequate.

You could use the tables on the previous sheet (the one accompanying the graph-sheet) to statistically validate the fit: scroll down until reaching the "Parameter-Estimates" and "Goodness-of-Fit tests" tables.

| Distribution | Goodness-of-Fit Tests |  |  |  |  |  |
| :--- | :--- | ---: | ---: | ---: | ---: | :---: |
|  | Residuals <br> Std | Kolmogorov-Smirnov |  |  | Cramer-von Mises |  |
|  | Statistic | $p$ Value | Statistic | $p$ Value |  |  |
| Exponential | 0.0333583 | 0.0648110 | $<.0001$ | 688.91 | $<.0001$ |  |
| Lognormal | 0.0504281 | 0.0878340 | $<.0001$ | 1574.35 | $<.0001$ |  |
| Lognormal (Shifted) | 0.0070425 | 0.0211673 | $<.0001$ | 30.71 | $<.0001$ |  |

## Example 2.2: Distribution mixture fitting

We now try to accommodate the behavior to the right of the origin in the "Agent service time" histogram by a mixture of distributions.

Via SEEStat return to the "Statistical Models (Summaries)" window, click "New Model", select "Distributions" and "Mixture fitting".


Open the "Options" tab. You can select a homogeneous or heterogeneous (mixture of various distributions) option. The former is the default. Select "Lognormal". Set the number of mixture components to 5, select chart type Polygon.


## Click "OK".



You observe an excellent fit (Red line). In particular, on the left side (near the origin), there are two components, accommodating the very short and short calls.

Going to the previous Excel sheet, to view the corresponding Tables (by scrolling down), one notes that the main component has a weight of $91 \%$ in the mixture-its role in the chart is to fit the part beyond 30 seconds, which it does very well.

| Parameter Estimates |  |
| :--- | ---: |
| Components | Mixing <br> Proportions <br> $(\%)$ |
| 1. Lognormal | 3.19 |
| 2. Lognormal | 3.55 |
| 3. Lognormal | 91.09 |
| 4. Lognormal | 1.83 |
| 5. Lognormal | 0.34 |

See Appendix A for a short explanation on Mixture fitting and the algorithms used for distribution fitting.

## Example 2.3: Survival analysis with smoothing of hazard rates

SEEStat supports several survival models. These are required, for example, in order to achieve insight into customers' (im)patience, namely the time customers are willing to wait prior to hanging up. Indeed, for those customers who got served, their waiting time provides only a lower bound on how long they are willing to wait-their (im)patience constitutes censored observations. One must thus "uncensor" the data to produce adequate estimates. To this end, we now use simple tools from survival analysis. These will produce hazard-rate functions, which provide natural statistical summaries of (im)patience.

Return, via SEEStat, to the "Statistical Models (Summaries)" window, click "New Model".

Select "Survival analysis" and "Survival Curve Estimate".


There are two variable tabs. The first tab "Censored time" is open. Select "Wait time (handled)": this corresponds to the waiting time of the customers who received service. Open the "Failure time" tab and select "Wait time (unhandled)": this corresponds to the waiting of customers who joined the queue but did not receive service (mainly due to abandonment, though there are sometimes other reasons such as system malfunction).


Open the "Options" tab. SEEStat supports several methods of smoothing, which are applicable to hazard rates and beyond. Select "Default" smoothing (which, this time, happens to be the method of HEFT).

See Appendix B for a reference list on smoothing methods.


From the tab "Select categories" select "Telesales".
Click "Dates". Select "April 2001" and on the tab "Days" select "Weekdays".
Click "OK".


A noticeable peak in the hazard rate indicates that there is a trigger for customers to abandon after about 50 seconds of waiting (which, based on our experience, could be the result of a voice-announcement at that time: such announcements, regardless of their content, "reminds" customers of their wait and thus increase their likelihood of abandonment).

## Example 2.4: Smoothing of intraday time series

Smoothing algorithms are available for several statistical models. We now demonstrate the application of smoothing on the data used in Example 1.2.

Return as usual to "Statistical Models (Summaries)", click "New Model", select "Time Series" and "Intraday". Select "Arrivals to queue". In "Options" tab select "Default" smoothing (this time, the default is the method of Cubic Splines).


Select "Scatter" as chart type.
In the "X Properties" tab, set resolution to 02:00 = 2 minutes.
Click "Dates", mark "Individual days" and select "September 2001".
In the "Days" tab, if not already selected, select (with "Ctrl" and click) all four Tuesdays of September.
Click "OK"


For this small resolution of 2 minutes, there is plenty of noise, but the smoothed data clearly identifies the regular pattern that was discovered before. (Note that the smoothed curves are computed with the minimal resolution for this variable, which is 30 seconds; the 2-minute resolution is only for display.)

Click "Output" on the main menu, then click "Modify Tables and Charts". Open the "Properties" tab, set resolution to 15 min and click "OK".


The Averaged Data (over 15 minutes) is now much closer to the smoothed curves, as expected.

## Part 3

Some additional interesting examples.

## Example 3.1: Queue regulated by a protocol \& announcements

Via SEEStat return to the "Statistical Models (Summaries)" window, click "New Model", then click the "Distributions" button. Three available distribution models appear. Select "Estimates". In the "Variables" tab select (using Ctrl) both "Wait time (unhandled)" and "Wait time (handled)".

In the "Options" tab select chart type Polygon. Click "Dates->", select December 2002, make sure the "Aggregated days" option is selected, and in "Days" select Weekdays. Click "<-Tables". In "Select Categories" select "Quick\&Reilly". Click "OK".


Both lines are periodical. To get a better focus, you will cut the chart on the left side.
Click "Output" on the main menu and then "Modify Tables and Charts". Open "Properties", set the low limit to 5 seconds.


Click "OK".


As you see, the Wait time (unhandled), in blue, peaks every 65 sec. The Wait time (handled), in red, peaks every 130 seconds. These interesting observations are yet to find their explanations. (Our experience suggests that peaks in the "Wait-time (unhandled)" are "psychological", for example a reaction of a customer to an announcement; and peaks in the "Wait-time (handled)" are "protocol-driven", for example a result of a priority upgrade.)

## Example 3.2: Queue length \& state-space collapse

Via SEEStat return to the "Statistical Models (Summaries)" window, click "New Model". Click the "Time Series" button and select "Intraday".
In the "Variables" tab select "Customers in queue (average)".
In the "Options" tab select smoothing "None" and chart type "Polygon".
In the "X Properties" tab select resolution 1 minute.
In "Select Categories" tab select (with Ctrl and click) Business and Platinum.
Click "Dates->", select "Dates totals only", select the 8 months from May 2002 to December 2002 and select Weekdays in the "Days" tab. Click "OK".


Platinum is a small-scale service. You will now normalize the chart in order to identify patterns.

Click "Output" on the main menu and then "Modify Tables and Charts". Open the "Options" tab and select Percent to mean. Click "OK".


Note the essentially overlapping patterns of the queue lengths of the two customer types. (This phenomenon is predicted by asymptotic analysis of queues in heavy traffic, where it is referred to as State-Space-Collapse.)

Exit SEEStat, either via the " X " on the top-right corner, or by clicking "Close SEEStat" in the Main menu. (Don't exit the terminal.)

## HomeHospital Data

Background: The data we rely on was collected at a large Israeli hospital. This hospital consists of about 1000 beds and 45 medical units. The data includes detailed information on patient flow throughout the hospital, over a period of several years (January 2004-October 2007). In particular, the data allows one to follow the paths of individual patients throughout their stay at the hospital, including admission, discharge, and transfers between hospital units. The data does not acknowledge resolutions within the ED or within wards.

$N_{d}$ - average number of patients that arrived per weekday, for period January 1, 2004-October, 31, 2007
$N_{y}$ - average number of patients that arrived per year, for years 2004, 2005, 2006, all days (for year 2007 data not fully completed; missing two months-November and December).
$N$ - average number of patients in ED/ED-to-Ward transfer/Wards, recorded at 12:00 per weekday, for period January 1, 2004-October, 31, 2007
$N_{X \text {-Ray }}$ - average number of patients in X-Ray at 10:00 per weekday, for period January 1, 2004-October, 31, 2007
LOS - length of stay in ED/ED-to-Ward transfer/Wards/X-Ray per weekday, for period January 1, 2004-October, 31, 2007

Reopen SEEStat 3.0 and select HomeHospital study.


## Part 4: Hospital

## Example 4.1: Arrivals - Average per one weekday over entire month

Click "Main" and "Statistical Models (Summaries)". Select "Time Series", then "Daily totals".
From the variables list select "Patient Arrivals at Hospital".
In the "Select Categories" tab, select all categories.
Click the "Dates->" button. Click the "Select all" button, open tab "Days" and select "Weekdays".
Click "OK".


We see a drop in the number of arrivals during July and August 2006. The reason for this phenomenon is the "second Lebanon war" which took place during that period and affected mostly the northern part of Israel in which the hospital is located. This could be verified via Calendar, in which special days (such as holidays) and special events are noted.

Click View-> Calendar. Mark "Individual days" and select July 2006. Open tab "Days".


Click "Months" tab and select August 2006. Open tab "Days".

## Part 5: Emergency Department

Example 5.1: Time by ED Internal state (sec.), or equivalently ED census Distribution during all 24 hours of the day

Return to the "Statistical Models (Summaries)" window.
Click the "New Model" button. Select "Distributions", then "Estimates".
In the "Variable" tab, select "Time by ED Internal state (sec.)".
In the "Select Categories" tab, select "Total".
In the "X Properties" tab change upper quantile limit to $\mathbf{1 0 0}$.


Click the "Dates->" button. Select "Dates totals only" and all months from January 2004 to October 2007, open the "Days" tab and select "All days".
Click "OK".


For example: 28 patients were in Internal ED during 38 minutes and 30 seconds (2.674\% from 24 hours: $0.02674 * 86400 \mathrm{sec}=2310 \mathrm{sec}$ ) between 00:00 and 24:00.

| Statistics |  |
| :--- | ---: |
|  | Time by <br> ED Internal <br> state <br> (sec.) |
| N | 120960000 |
| N(average per <br> day) | 86400 |
| Mean | 23.63 |
| Standard <br> Deviation | 10.7 |
| Variance | 114.4 |
| Median | 22 |
| Minimum | 0 |
| Maximum | 70 |

We observe that the distribution has an unusual shape; it is skewed to the left, has a light right tail and has two local peaks. We shall now further investigate it via Example 5.2.

Example 5.2: Time by ED Internal state (sec.), or equivalently ED census Distribution during each of the $\mathbf{2 4}$ hours per day

Return to the "Statistical Models (Summaries)" window.
Click the "<-Tables" button.
In the "Select Categories" tab, select (with shift key) all categories except "Total".
Click "OK".


For example: 28 patients were in Internal ED during 3 minutes and 17 seconds (5.49\% from 1 hour: 0.0549*3600 sec=197 sec) between 11:00 and 12:00

| Statistics |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \\ & \dot{1} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & O \\ & \dot{O} \\ & \dot{O} \\ & \dot{i} \\ & \dot{O} \\ & \dot{O} \\ & \dot{O} \\ & \dot{O} \end{aligned}$ |  | $\begin{aligned} & \dot{O} \\ & \dot{0} \\ & \dot{0} \\ & \dot{0} \\ & \dot{o} \\ & \dot{0} \\ & \dot{\theta} \end{aligned}$ | $\begin{aligned} & \dot{O} \\ & \dot{O} \\ & \dot{0} \\ & \vdots \\ & \dot{o} \\ & \dot{0} \\ & \dot{0} \end{aligned}$ |  | $\begin{aligned} & \dot{O} \\ & \dot{\circ} \\ & \dot{\circ} \\ & \dot{1} \\ & \dot{o} \\ & \dot{\circ} \\ & \dot{\theta} \\ & \hline 0 \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \dot{O} \\ & \dot{O} \\ & \dot{N} \\ & \dot{N} \\ & \dot{O} \\ & \dot{O} \\ & \dot{d} \end{aligned}$ |  |  |
| z | $\begin{aligned} & \hline \text { oi } \\ & \text { ob } \\ & \text { on } \end{aligned}$ | $\begin{array}{\|l\|} \hline 8 \\ \hline 8 \\ \text { ob } \\ \hline \end{array}$ | $\begin{array}{\|l\|l} \hline 8 \\ \text { oid } \\ \text { O } \\ \text { in } \end{array}$ | $\begin{array}{\|c\|} \hline 0 \\ 0 \\ 0 \\ \text { B } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 8 \\ & \text { ob } \\ & \hline 0 \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { oin } \\ \text { Ob } \\ \text { on } \end{array}$ | $\begin{array}{\|l\|} \hline 8 \\ \hline 8 \\ \text { O} \\ \hline \end{array}$ | $\begin{array}{\|l\|l} \hline \text { oin } \\ \text { Ob } \\ \text { on } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 0 \\ & \text { O} \\ & \text { in } \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { oin } \\ \text { Ob } \\ \text { on } \end{array}$ | $\begin{array}{\|l\|} \hline 8 \\ \hline 8 \\ \text { O} \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \text { oin } \\ \text { Ob } \\ \text { on } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 8 \\ & \text { O} \\ & \text { in } \end{aligned}$ | $\begin{array}{\|l\|} \hline 8 \\ \text { ob } \\ \text { od } \\ \text { in } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 8 \\ & \text { ob } \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { oin } \\ \text { Ob } \\ \text { ob } \end{array}$ | $\begin{array}{\|l\|} \hline \text { oi } \\ \text { Ob } \\ \text { in } \end{array}$ | $\begin{array}{\|l\|} \hline 8 \\ \text { ob } \\ \text { od } \\ \text { in } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 8 \\ & \text { o } \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { oin } \\ \text { Ob } \\ \text { on } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 0 \\ & \text { O } \\ & \text { in } \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { oin } \\ \text { Ob } \\ \text { on } \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 8 \\ & \text { ob } \\ & \hline 0 \end{aligned}$ | － |
|  | ষ্ণ | ిo্ల | 若 | \|ob | ষ্টি | $\stackrel{\stackrel{\rightharpoonup}{0}}{ }$ | 若 | $\stackrel{\rightharpoonup}{\mathbf{O}}$ | $\stackrel{\stackrel{\rightharpoonup}{\mathrm{e}}}{\mathbf{2}}$ | $\stackrel{\rightharpoonup}{\mathbf{O}}$ | 若 | $\stackrel{8}{0}$ | 若 | $\stackrel{8}{0}$ | ৪ | $\stackrel{\otimes}{\mathrm{O}}$ | $\stackrel{\stackrel{\rightharpoonup}{\mathrm{e}}}{\mathbf{~}}$ | $\stackrel{\text { O}}{0}$ | ষ্ণ | $\stackrel{\stackrel{0}{0}}{ }$ | $\stackrel{\stackrel{\rightharpoonup}{\mathrm{e}}}{\mathbf{2}}$ | $\stackrel{\stackrel{0}{0}}{ }$ | 若 | \％ |
|  | $\begin{gathered} \text { ® } \\ \text { ín } \end{gathered}$ | $\begin{aligned} & \text { N } \\ & \underset{\sim}{\infty} \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \underset{\sim}{0} \\ \hline \end{array}$ | $\left.\begin{array}{\|c\|} \otimes \infty \\ \underset{寸}{\mid} \end{array} \right\rvert\,$ | $$ | $$ | $\begin{array}{\|l\|} \hline \infty \\ \underset{A}{\prime} \end{array}$ | $\begin{array}{\|c\|c\|c\|c\|c\|c\|} \substack{n} \end{array}$ | $\pm$ | $$ | $\begin{array}{\|l} \hline \underset{\sim}{\circ} \\ \hline \end{array}$ | $$ | $\begin{array}{\|c} \underset{\sim}{\underset{\sim}{\infty}} \\ \hline \end{array}$ | $$ | $\begin{aligned} & \hline \stackrel{\circ}{\circ} \\ & \stackrel{\circ}{\mathrm{B}} \end{aligned}$ | $\stackrel{\sim}{m}$ | － | Nóm | $\begin{aligned} & \stackrel{1}{7} \\ & \stackrel{\rightharpoonup}{6} \end{aligned}$ | $\begin{aligned} & \text { O. } \\ & \hline \text { B } \end{aligned}$ | $\begin{aligned} & \mathrm{J} \\ & \underset{ल}{\prime} \end{aligned}$ | $$ | $\begin{array}{\|l\|l} \hline \stackrel{\infty}{8} \\ \hline 8 . \end{array}$ | $\begin{aligned} & \stackrel{\circ}{\text { ®N }} \end{aligned}$ |
|  | $\stackrel{\circ}{\sim}$ | $\stackrel{\circ}{\circ}$ | －7 | ¢ | i | $\stackrel{\circ}{寸}$ | $\stackrel{\infty}{\dot{+}}$ | $\stackrel{\circ}{\dot{+}}$ | ¢ | － | $\stackrel{\text { ¢ }}{6}$ | $\stackrel{\circ}{\sim}$ | － | $\stackrel{\infty}{\infty}$ | $\stackrel{\circ}{\infty}$ | ने | $\stackrel{\circ}{\circ}$ | $\stackrel{\infty}{\circ}$ | $$ | $0$ | $\begin{aligned} & \hline \stackrel{\circ}{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{\sim}{\circ}$ | $\stackrel{0}{\circ}$ |
|  | $\begin{aligned} & \text { oi } \\ & \dot{\infty} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { ơ } \\ & \text { ⿷匚 } \end{aligned}$ | $\begin{array}{\|l\|l} \hline \stackrel{\rightharpoonup}{\mathrm{m}} \\ \hline \end{array}$ | $\left\|\begin{array}{c} \stackrel{\sim}{\mathrm{a}} \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{aligned} & \text { N } \\ & \text { Ni } \end{aligned}$ | $\begin{array}{\|l\|l} \hline \text { Br } \\ \text { di } \end{array}$ | $\begin{array}{\|l\|} \stackrel{\circ}{\sim} \\ \hline \end{array}$ | $$ | $\begin{aligned} & \stackrel{ֻ}{\mathrm{~N}} \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \stackrel{\circ}{\circ} \\ \stackrel{y}{*} \end{array}$ | $\begin{aligned} & \text { Ø } \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \text { co } \\ & \dot{\circ} \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l} \hline \stackrel{\text { ® }}{1} \end{array}$ | $\begin{array}{\|l\|l\|} \hline \stackrel{y y}{\infty} \\ \infty \\ \hline \end{array}$ | $\begin{aligned} & \text { N్ } \\ & \text { in } \end{aligned}$ | $\stackrel{\stackrel{y}{\infty}}{\substack{\infty}}$ | $\begin{aligned} & \stackrel{\infty}{6} \\ & \underset{\sim}{\circ} \end{aligned}$ | $\stackrel{\stackrel{\circ}{\circ}}{\stackrel{\circ}{\circ}}$ | $\begin{aligned} & \text { M } \\ & \dot{d} \\ & \text { den } \end{aligned}$ |  | $\begin{aligned} & 7 \\ & \vec{~} \\ & \text { - } \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \\ & \text { oun } \end{aligned}$ | $\begin{gathered} \text { 凹゙ } \\ \text { む゙ } \end{gathered}$ | $\begin{aligned} & \text { d } \\ & \text { di } \end{aligned}$ |
|  | $\stackrel{1}{2}$ | $\stackrel{\infty}{\sim}$ | $\stackrel{\square}{\square}$ | $\pm$ | $\pm$ | $\pm$ | ন | $\pm$ | $\pm$ | $\stackrel{\square}{-}$ | $\stackrel{1}{2}$ | $\stackrel{\sim}{\sim}$ | $\stackrel{\infty}{\sim}$ | ¢ | \％ | $\vec{m}$ | － | － | \％ | ¢ | － | － | ¢ | $\stackrel{\sim}{\sim}$ |
|  | － | $\bigcirc$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\rightarrow$ | $\checkmark$ | $\checkmark$ | $\rightarrow$ | $\sim$ | ＊ | ＊ | ＊ | $\checkmark$ | ＊ | $m$ | ～ | $\sim$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
|  | ก | $\stackrel{\text { ¢ }}{ }$ | ₹ | \％ | ¢ | d | ¢ | \％ | \％ | ¢ | g | ๗ | ¢ | $\stackrel{8}{8}$ | ® | ก | ¢ | ¢ | ¢ | d | ¢ | $\bigcirc$ | 8 | $\square$ |

Click＂Output＂on the main menu and then＂Modify Tables and Charts＂．
In the＂Options＂tab，under the＂Convert to＂select＂Frequencies＂and select chart type ＂Polygon＂．
Click＂OK＂．


We observe that the distribution of the number of patients in the ED at the time of the day $t$, (for $t=0,1, \ldots, 23$ ) is normal, with mean and variance that vary over time (this was confirmed by the appropriate statistical tests, see Example 5.4). Using these figures, we identify three main patterns that compose the distribution found in Example 5.1: (1) From 02:00 until 09:00, where the average number of patients is around 15; (2) From 12:00 until 22:00, where the average number of patients is about 35; and (3) The rest of the day (09:0012:00, 22:00-02:00), when the distribution shifts from one group to the other.
This is further observed in Example 5.3.

## Example 5.3: Number of patients in Internal ED - Average per 10-minute intervals, only on Mondays during 2005

Return to the "Statistical Models (Summaries)" window.
Click the "New Model" button. Select "Time Series", then "Intraday".
In the "Variable" tab, select "Number of Patients in Emergency Department (average)". In the "Select Categories" tab, select "Emergency Internal Medicine Unit".
Click the "Dates->" button.
Mark "Individual days for aggregated day".
Select months from June 2005 to December 2005.


Open the "Days" tab and select "Mondays". Click "OK".


## Example 5.4: Time by ED Internal state (sec.) - Fitting distribution during "evening" hours, on Mondays, 2005

Return to the "Statistical Models (Summaries)" window.
Click the "New Model" button. Select "Distributions", then "Fitting". In the "Variable" tab, select "Time by ED Internal state (sec.) (by Day_time2)". In the "Options" tab select Normal distribution. In the "Select Categories" tab, select "[13:00-23:00)". Click the "Dates->" button.
Mark "Dates totals only". Select months from January 2005 to December 2005.
Open tab "Days" and select "Mondays".
Click "OK".


| Statistics |  |
| :--- | ---: |
|  | Time by ED Internal state (sec.) |
| N | 1836000 |
| N (average per day) | 36000 |
| Mean | 32.73 |
| Standard |  |
| Deviation | 6.029 |
| Variance | 36.35 |
| Median | 32 |


| Parameters for |  |
| :--- | ---: |
| Normal Distribution |  |
| Parameter | Estimate |
| mu | 32.73 |
| sigma | 6.03 |
| mean | 32.73 |
| std | 6.029 |


| Goodness-of-Fit Tests for |  |  |  |
| :--- | ---: | ---: | :--- |
| Normal Distribution |  |  |  |
| Test | Statistic | DF | $p$ <br> Value |
| Residuals Std | 0.0209 |  |  |
| Kolmogorov-Smirnov | 0.0587 |  | $<.0001$ |
| Cramer-von Mises | 801.9579 |  | $<.0001$ |
| Anderson-Darling | 4532.0570 |  | $<.0001$ |
| Chi-Square | $>1000$ | 41 | $<.0001$ |

Note: probability density function formulas for 50 continuous distributions can be found under the "Options" tab, by clicking the "Distributions Description" button.



Statistical Continuous Distributions Names in SEEStat Interface


## Part 6: Medical Wards

## Example 6.1: LOS in Internal Medicine (in days) - Distribution fitting

Return to the "Statistical Models (Summaries)" window.
Click the "New Model" button. Select "Distributions", then "Fitting".
In the "Variable" tab, select "Patient length of stay in Ward (days) (by ward_department)".
In the "Options" tab, under the "Convert to" select "Relative frequencies".
In the "Select Categories" tab, select "Internal Medicine A".
In the "X Properties" tab, under "Range to Display" change the Upper Quantile limit to 97.5. Click the "Range to Compute" button and choose "Select Range", change the Low Limit to 1 and the Upper Quantile to 100.
Click the "Dates->" button.
Mark "Dates totals only". Select months from January 2004 to October 2007 (using the shift key).
Open the "Days" tab and select "All days".
Click "OK".


When considering daily resolutions, the Log-Normal distribution turns out to fit the LOS distribution well.

## Example 6.2: LOS in Internal Medicine (in hours) - Distribution

Return to the "Statistical Models (Summaries)" window.
Click the "New Model" button. Select "Distributions", then "Estimates".
In the "Variable" tab, select " Patient length of stay in Ward (hours) (by ward_department)".
In the "Select Categories" tab, select "Internal Medicine A".
In the "X Properties" tab change the Upper Quantile limit to 95.
Click the "Dates->" button.
Mark "Dates totals only". Select months from January 2004 to October 2007.
Open the "Days" tab and select "All days".
Click "OK".


In the 1-hour resolution we observe a completely different LOS distribution, with peaks that are periodically 24 hours apart. The reason for this phenomena is the discharge procedure that is performed over "batches" of patients and, hence, takes a few hours. This results in a very low variance of the discharge time, as most patients are released between $3 p m$ and $4 p m$ (see Example 6.3).

## Example 6.3: Patient Discharges from Ward - Intraday time series

Return to the "Statistical Models (Summaries)" window; click "New Model", select "Time Series" and "Intraday".
In the "Variables" tab, select "Patient Discharges from Ward".
In the "Select Categories" tab, select
"Department of Internal Medicine",
"Department of Orthopedics",
"Department of General Surgery",
"Department of Cardiac Surgery",
"Department of Maternity",
"Department of Gynecology".

Open the "X Properties" tab and change the low limit to 08:00.
Click the "Dates->" button. Mark "Dates totals only" and select months from January 2004 to October 2007. Open the "Days" tab and select "Weekdays". Click "OK".


For additional reading on a data-based perspective on patient flow in hospitals, using the HomeHospital database, see the following paper: "On patient flow in hospitals: A data-based queueing-science perspective" by Mor Armony, Shlomo Israelit, Avishai Mandelbaum, Yariv N. Marmor, Yulia Tseytlin, and Galit B. Yom-Tov. Stochastic Systems, 5, (2015), 146-194. http://ie.technion.ac.il/serveng/References/Short_Patient_flow_main.pdf or the extended version on:
http://ie.technion.ac.il/serveng/References/Patient_flow_main_EV.pdf

## Addendum

What you have experiences is only the beginning of what SEEStat can offer.
In fact, SEEStat also has "relatives" that could take you way further. For example, consider SEEGraph which creates semi-automatically data-animations. Examples are in
https://www.youtube.com/watch?v=1A6-jzS_scI
https://www.youtube.com/watch?v=jx3UUQCPODE
https://www.youtube.com/watch?v=-ik5kA7aLGg
where you can "observe" customers and agents in call centers, doctors and nurses in hospital, and more.


## Appendix A: Mixture Fitting and Distribution Fitting

## Mixture Fitting

The mixture distribution function:

$$
p(x)=\sum_{j=1}^{k} w_{j} \varphi\left(x ; \theta_{j}\right) ; \quad \sum_{j=1}^{k} w_{j}=1 ; \quad w_{j}>0
$$

$\theta_{j}$ - distribution parameters (shape, scale or location parameters)
$\varphi$ - given distribution function (Normal, Lognormal, Gamma, Exponential, Weibull or Invers Gaussian)
$k$ - given number of components of the mixture
$w_{j}$ - weighting coefficient or mixing proportions

Task: To find the optimal $w_{j}$ and $\theta_{j}$ for sample X and given $k$ and $\varphi$.

## Solution:

- Specify initial values for $w_{j}$ by using grid.
- For each point in a grid calculate mean, standard deviation (specify initial values for $\theta_{j}$ ) and objective function (sum of squares of difference between empirical and fitted cumulative distribution functions).
- Select points with smallest values of objective function (best points).
- For each best point compute fitted distribution by using optimization algorithm.
- The best fit with smallest value of objective function is selected.


## Distribution Fitting

Algorithms of distribution fitting (SEEStat software):

1. Maximum likelihood estimates.
(11 distributions)
2. L-moments estimates (linear combination of order statistics).
(31 distributions)
3. Maximization of likelihood function.
(7 distributions)
4. Optimization of the object function (sum of squares of differences between empirical and fitted cumulative distribution functions)
(1 distribution)

## Appendix B: Smoothing References

## Heft: Hazard estimation with flexible tails

[1] Charles Kooperberg, Charles J. Stone and Young K. Truong (1995). Hazard regression. Journal of the American Statistical Association, 90, 78-94.
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## Loess: Local polynomial regression fitting

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## Muhaz: Estimate hazard function from right-censored data

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## Supsmu: Friedman's SuperSmoother

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## Pspline: Fit a polynomial smoothing spline of arbitrary order

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## Bspline: Fits a cubic smoothing spline to the supplied data

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## dpill: Select a bandwidth for local linear regression

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[14] M.P. Wand and M.C. Jones (1995). Kernel Smoothing. Chapman and Hall, London.

## dpih: Select a histogram bin width

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## dpik: Select a bandwidth for kernel density estimation

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[19] M.P. Wand and M.C. Jones (1995). Kernel Smoothing. Chapman and Hall, London.

## locpoly: Estimate functions using local polynomials

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## density: Kernel Density Estimation

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